AQA

A-level
FURTHER MATHEMATICS
7367/1

Paper 1

Mark scheme

June 2022

Version: 1.0 Final Mark Scheme

226A7367/1/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	2.2a	B1	$\frac{2\pi}{3}$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Ticks correct answer	1.1b	B1	$\cos\left(\frac{8\pi}{13}\right)+\mathrm{i}\sin\left(\frac{8\pi}{13}\right)$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Ticks correct answer	2.2a	B1	$-\operatorname{sech} x \tanh x$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4	Circles correct answer	2.2a	B1	36
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	States complex conjugate	1.1b	B1	$z = -\frac{3}{2} - \mathrm{i}\frac{\sqrt{11}}{2}$ is another root Sum of complex roots = –3 Product of complex roots = 5 So $z^2 + 3z + 5$ is a factor of $z^4 - 3z^3 - 5z^2 + kz + 40$ The other quadratic factor is $z^2 - 6z + 8$ $= (z-2)(z-4)$ The other roots are $2, 4, -\frac{3}{2} - \mathrm{i}\frac{\sqrt{11}}{2}$
	Obtains a quadratic factor or Obtains sum and product of their two complex roots or Substitutes given root or their conjugate into the quartic	1.1a	M1	
	Obtains $z^2 + 3z + 5$ or Obtains $k = -6$	1.1b	A1	
	Forms a second quadratic or Solves the quartic equation with their value of k	1.1a	M1	
	Obtains 2 and 4	1.1b	A1	
	Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Deduces correct solution (ft from their exactly two distinct real roots)	2.2a	B1F	$2 < x < 4$
	Total		**1**	

	Question total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Recalls exponential definition of tanh	1.2	B1	Let $y = \tanh^{-1} x$ Then $x = \tanh y$ $x = \dfrac{e^y - e^{-y}}{e^y + e^{-y}}$ $x(e^y + e^{-y}) = e^y - e^{-y}$ $e^y(x-1) + e^{-y}(x+1) = 0$ $e^{2y}(x-1) + (x+1) = 0$ $e^{2y} = \dfrac{1+x}{1-x}$ $2y = \ln\left(\dfrac{1+x}{1-x}\right)$ $\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ as required.
	Multiplies by their denominator and multiplies by e^y (or e^x)	3.1a	M1	
	Obtains $e^{2y} = \frac{1+x}{1-x}$ or Obtains $e^{2x} = \frac{1+y}{1-y}$ OE	1.1b	A1	
	Completes a rigorous argument to show the required result	2.1	R1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Uses appropriate hyperbolic identity (condone sign errors) to obtain an equation in one hyperbolic function or substitutes exponential form, condone error in sum/difference of exponential terms	1.1a	M1	$20(1 - \tanh^2 x) - 11\tanh x = 16$ $0 = 20\tanh^2 x + 11\tanh x - 4$ $(5\tanh x + 4)(4\tanh x - 1) = 0$ $\tanh x = -\frac{4}{5}$ or $\frac{1}{4}$ $x = \tanh^{-1}\left(-\frac{4}{5}\right) = \frac{1}{2}\ln\left(\frac{1}{9}\right)$ or $x = \tanh^{-1}\left(\frac{1}{4}\right) = \frac{1}{2}\ln\left(\frac{5}{3}\right)$
	Solves their quadratic equation to obtain two solutions in $\tanh x$ or quadratic in e^{2x} to obtain two solutions or quartic in e^x to obtain at least two solutions	1.1a	M1	
	Obtains $\tanh x = -\frac{4}{5}$ and $\frac{1}{4}$ OE or Obtains $e^{2x} = \frac{5}{3}$ and $\frac{1}{9}$ OE or Obtains $e^x = \frac{\sqrt{15}}{3}$ and $\frac{1}{3}$ OE	1.1b	A1	
	Obtains correct solutions (any correct exact log form) ISW	1.1b	A1	
	Total		**4**	

	Question total		**8**	

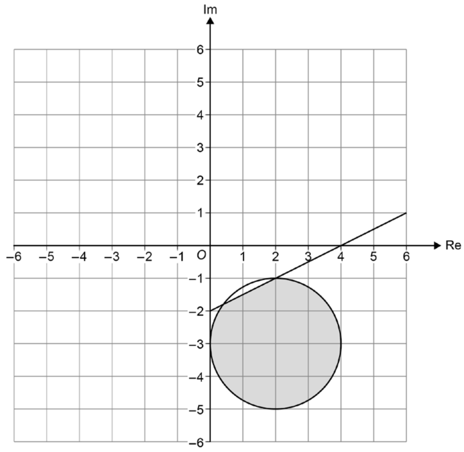
Q	Marking instructions	AO	Marks	Typical solution
7(a)(i)	Expands $\lvert\mathbf{M}\rvert$ to get a linear expression in k, using any row or column.	1.1a	M1	$\lvert\mathbf{M}\rvert = (12-3k-3)-7(6-k-1)-3(9-6)$ $= 4k-35$ Cofactors are $\begin{bmatrix} 9-3k & k-5 & 3 \\ -23 & 5 & 4 \\ 7k+25 & -k-10 & -15 \end{bmatrix}$ $\mathbf{M}^{-1} = \frac{1}{4k-35}\begin{bmatrix} 9-3k & -23 & 7k+25 \\ k-5 & 5 & -k-10 \\ 3 & 4 & -15 \end{bmatrix}$
	Obtains matrix of minors/cofactors with at least four correct elements PI transposed form. Condone overall sign error on each element.	1.1b	B1	
	Obtains matrix of minors/cofactors with at least seven correct elements PI transposed form. Condone overall sign error on each element.	1.1b	B1	
	Obtains correct matrix of minors/cofactors PI transposed form. Condone overall sign error on each element.	1.1b	B1	
	Obtains fully correct, simplified answer	1.1b	A1	
	Total		**5**	

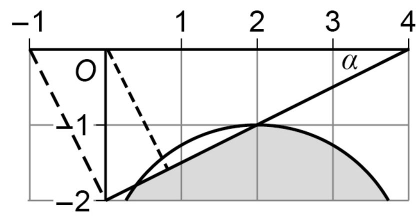
Q	Marking instructions	AO	Marks	Typical solution
7(a)(ii)	Obtains correct answer for their linear expression for $\lvert\mathbf{M}\rvert$	1.1b	B1F	$k \neq \frac{35}{4}$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Obtains their correct $\mathbf{M}^{-1}$,need not be simplified.	3.1a	B1F	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{15}\begin{bmatrix} -6 & -23 & 60 \\ 0 & 5 & -15 \\ 3 & 4 & -15 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 1 \end{bmatrix}$
	Forms their product $\mathbf{M}^{-1}\begin{bmatrix} 6 \\ 3 \\ 1 \end{bmatrix}$	1.1a	M1	$= \frac{-1}{15}\begin{bmatrix} -45 \\ 0 \\ 15 \end{bmatrix}$
	Obtains $x = 3$, $y = 0$, $z = -1$ ACF CSO	1.1b	A1	$= \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$ $x = 3$, $y = 0$, $z = -1$
	Total		**3**	

	Question total		**9**	

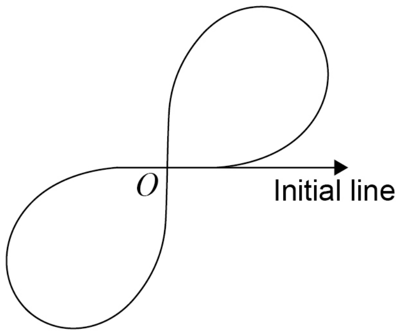
Q	Marking instructions	AO	Marks	Typical solution
8(a)	Deduces correct gradient or intercept PI	2.2a	M1	Gradient = $\frac{1}{2}$ Line passes through (0, –2) $y=\frac{1}{2}x-2$
	Obtains correct equation with y as the subject	1.1b	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)	Draws a half line from –2i passing through 4. Condone full line ft their linear equation in part **(a)**	1.1b	B1F	
	Draws circle or arc of a circle, with centre at 2–3i or radius 2	2.2a	M1	
	Draws circle or arc of a circle, centre at 2–3i and radius 2	1.1b	A1	
	Correct region indicated	2.2a	A1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
8(c)(i)	Identifies the point in their region nearest to the origin. For example draws the perpendicular from the half-line to the origin or Finds $y = -2x$	3.1a	B1F	 $\sin\alpha = \frac{\sqrt{5}}{5}$ $\frac{\lvert z_1\rvert}{4} = \sin\alpha$ $\lvert z_1\rvert = \frac{4\sqrt{5}}{5}$
	Finds the distance between their valid point and the origin For example uses $\sin\left(\tan^{-1}\frac{1}{2}\right)$ or Finds the distance between the origin and the point of intersection of the lines $y = \frac{1}{2}x - 2$ and $y = -2x$	3.1a	M1	
	Obtains correct exact value of $\lvert z_1\rvert$	1.1b	A1	
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
8(c)(ii)	Uses their values from part **(c)(i)** to obtain value of a or b	1.1a	M1	$a = \lvert z_1\rvert \sin\alpha$ $\quad b = -\lvert z_1\rvert\cos\alpha$ $= \frac{4\sqrt{5}}{5}\times\frac{\sqrt{5}}{5}$ $\quad = \frac{-4\sqrt{5}}{5}\times\frac{2\sqrt{5}}{5}$ $= \frac{4}{5}$ $\quad = \frac{-8}{5}$ $z_1 = \frac{4}{5} - \frac{8}{5}\mathrm{i}$
	Obtains correct value of z_1 CSO	1.1b	A1	
	Total		**2**	

	Question total		**11**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Draws at least one loop in the correct place	1.1b	B1	
	Draws both loops correctly (approx equal size) and no others	1.1b	B1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)	Criticises limits of integration used by Roberto PI	2.3	M1	Roberto has used incorrect limits of integration. He should only have included values of θ which make $\sin 2\theta$ positive, because r^2 must be positive
	Explains what is wrong with Roberto's range of values, including reference to $r^2 \geq 0$	2.4	R1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)	Forms an expression for an area using valid limits PI by 9/4, 9/2 or 9	2.2a	M1	$A = \frac{1}{2}\int_{-\pi}^{-\frac{\pi}{2}} 9\sin 2\theta \, d\theta + \frac{1}{2}\int_{0}^{\frac{\pi}{2}} 9\sin 2\theta \, d\theta$ $= \int_{0}^{\frac{\pi}{2}} 9\sin 2\theta \, d\theta$ by symmetry $A = \left[-\frac{9}{2}\cos 2\theta\right]_0^{\frac{\pi}{2}}$ $= -\frac{9}{2}(\cos\pi - \cos 0)$ $= 9$
	Obtains correct answer	1.1b	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(d)	Deduces at least one correct value of θ	2.2a	M1	Max. value of $r = 3$ For r maximum, $\sin 2\theta = 1$ $2\theta = \frac{\pi}{2}, -\frac{3\pi}{2}$ $\theta = \frac{\pi}{4}, -\frac{3\pi}{4}$ $P\left(3, \frac{\pi}{4}\right)$ and $Q\left(3, -\frac{3\pi}{4}\right)$
	Obtains both correct solutions (not $r = -3$) Condone r and θ transposed. accept $\frac{5\pi}{4}$ etc OE decimals to 3sig fig or better	1.1b	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(e)(i)	Finds cartesian coordinates or position vector of their P, ACF	1.1b	B1F	Cartesian coordinates of *P* are $\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$ $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{3\sqrt{2}}{2} \\ \frac{3\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} \frac{9\sqrt{2}}{2} \\ \frac{3\sqrt{2}}{2} \end{bmatrix}$ For *P'*, $r^2 = \frac{81}{2} + \frac{9}{2} = 45 \Rightarrow r = 3\sqrt{5}$ $\tan\theta = \frac{1}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{3}\right)$ $P'\left(3\sqrt{5}, \tan^{-1}\left(\frac{1}{3}\right)\right)$
	Multiplies their cartesian position vector by matrix M, must be Mv, to obtain an image	3.1a	M1	
	Obtains their correct value for r or θ, ACF, for their image	1.1a	M1	
	Obtains their correct answer, either exact or to at least 2 sig fig AWRT (6.7, 0.32) Must have M1M1	1.1b	A1F	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
9(e)(ii)	Explains correctly why the area is unchanged	2.4	E1	Det M = 1 Area enclosed by C2 = (Area enclosed by C1) × det M = 9 × 1 = 9
	Obtains their correct area from their part **(c)** and their det	2.2a	B1F	
	Total		**2**	

	Question total		**14**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)	Obtains correct normal vector to the plane	2.2a	B1	Normal to plane $\mathbf{n} = \begin{bmatrix} 4 \\ p \\ 5 \end{bmatrix}$
	Obtains correct expression for $\overrightarrow{AB}$ or $\overrightarrow{BA}$	1.2	B1	Let $\overrightarrow{AB} = \mathbf{c}$ then $\mathbf{c} = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$
	Obtains their correct scalar (or vector) product	1.1b	B1F	$\mathbf{n} \cdot \mathbf{c} = 13 - 5p$ $\lvert\mathbf{c}\rvert = \sqrt{30}$ and $\lvert\mathbf{n}\rvert = \sqrt{41 + p^2}$
	Uses scalar (or vector) product to obtain an equation in p	3.1a	M1	As α is acute, $\sin\alpha = \cos\theta = \frac{\sqrt{15}}{75}$
	Forms an equation in p, by squaring and removing any rational functions Eg $82 + 2p^2 = 5^2(13 - 5p)^2$	1.1a	M1	Hence $\left\lvert \frac{13 - 5p}{\sqrt{30}\sqrt{41 + p^2}} \right\rvert = \frac{\sqrt{15}}{75}$
	Solves quadratic and selects correct answer, discarding the other root Condone lack of modulus sign in the working	2.1	R1	$82 + 2p^2 = 5^2(13 - 5p)^2$ $623p^2 - 3250p + 4143 = 0$ $p = 3$ or $p = \frac{1381}{623}$ $p \in \mathbb{Z}$ $\therefore p = 3$
	Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)	Recognises need to divide constant term of the plane equation by $\lvert\mathbf{n}\rvert$	1.1a	M1	$\lvert\mathbf{n}\rvert = 5\sqrt{2}$ Distance is $\frac{9}{5\sqrt{2}} + \frac{5}{5\sqrt{2}} = \frac{7\sqrt{2}}{5} = 1.98$ cm
	Finds correct distance for their p, exact or decimal at least 2sf (condone 2) Condone missing units	1.1b	A1F	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)	Obtains correct equation of the line through *A* & *A*' for their p Condone lack of $\mathbf{r} =$	3.1a	B1F	$\mathbf{r} = \begin{bmatrix} 7 \\ 2 \\ -3 \end{bmatrix} + \mu \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$ At Π_1 $4(7+4\mu)+3(2+3\mu)$ $+5(-3+5\mu)=9$ $\mu = \frac{-1}{5}$ At image point $\mu = \frac{-2}{5}$ $\left(\frac{27}{5}, \frac{4}{5}, -5\right)$
	Forms an equation to find the value of μ for their line Condone use of Π_2	3.1a	M1	
	Doubles their value of μ and uses it to find image point for their line Condone use of Π_2	3.2a	M1	
	Obtains correct coordinates for their p Do not accept position vector Do not condone use of Π_2	1.1b	A1F	
	Total		**4**	

	Question total		**12**	

Q	Marking instructions	AO	Marks	Typical solution
11(a)	Forms equilibrium force equation with three correct terms. Condone sign errors	3.1b	B1	In equilibrium position $7e_A = 9e_B + 0.32g\sin 30$ $1.2 = e_A + e_B$ $e_A = 0.775$, $e_B = 0.425$
	Forms at least one correct expression in x for the tension ie $9(e_B - x)$ or $7(e_A + x)$ Condone their incorrect e_A or e_B	1.1a	B1F	After release $9(e_B - x) + 0.32g\sin 30 - 7(e_A + x) = 0.32\ddot{x}$ $9(0.425 - x) + 0.32g\sin 30 - 7(0.775 + x) = 0.32\ddot{x}$ $3.825 - 9x + 1.6 - 5.425 - 7x = 0.32\ddot{x}$ $-16x = 0.32\ddot{x}$ $\ddot{x} + 50x = 0$ as required
	Forms general force equation with four terms (with at least two terms correct). Condone "a" for $\ddot{x}$ Condone sign errors on the terms Condone their incorrect e_A or e_B	3.1b	M1	
	Forms correct force equation. Can be in terms of e_A & e_B Condone "a" for $\ddot{x}$ Condone their incorrect e_A or e_B	1.1b	A1F	
	Constructs a rigorous argument to show the required result	2.1	R1	
	Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
11(b)(i)	Obtains fully correct 2nd order DE or auxiliary equation, any correct form. Condone $-k$ Allow m instead of 0.32	2.2a	B1	$9(0.425-x)+0.32g\sin 30-7(0.775+x)-k\dot{x}=0.32\ddot{x}$ $0.32\ddot{x}+k\dot{x}+16x=0$ $0.32\lambda^2+k\lambda+16=0$
	Sets up $b^2-4ac=0$ from their 2nd order DE or Auxiliary Equation	1.2	M1	Critical Damping so: $b^2-4ac=0$ $k^2-4\times 0.32\times 16=0$
	Obtains correct value of k from correct working	2.1	R1	$k^2=\dfrac{512}{25}$ $k=\dfrac{16\sqrt{2}}{5}$
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
11(b)(ii)	Obtains correct solution from their three term Auxiliary Equation	3.1a	M1	$0.32\lambda^2+\dfrac{16\sqrt{2}}{5}\lambda+16=0$ $\lambda=-5\sqrt{2}$ twice
	Obtains correct solution of their three term differential equation	1.1b	A1F	$x=Ae^{-5\sqrt{2}\,t}+Bte^{-5\sqrt{2}\,t}$ $x=0.2, t=0\Rightarrow A=0.2$
	Uses $x=0.2$ when $t=0$ to obtain correct A	3.3	B1	$\dot{x}=-5\sqrt{2}Ae^{-5\sqrt{2}\,t}+Be^{-5\sqrt{2}\,t}-5\sqrt{2}Bte^{-5\sqrt{2}\,t}$ $\dot{x}=0, t=0\Rightarrow B=\sqrt{2}$
	Sets their correct $\dot{x}=0$ when $t=0$	3.3	M1	$x=0.2e^{-5\sqrt{2}\,t}+\sqrt{2}te^{-5\sqrt{2}\,t}$
	Obtains their correct B Ft from $\lambda^2+\dfrac{16\sqrt{2}}{5}\lambda+50=0$	1.1b	A1F	
	Completes a rigorous argument to obtain the correct result	2.1	R1	
	Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
11(b)(iii)	Forms an equation to find the value of t at the maximum speed Must start from a damped harmonic model	3.1a	M1	$\dot{x} = -\sqrt{2}e^{-5\sqrt{2}\,t} + \sqrt{2}e^{-5\sqrt{2}\,t} - 10te^{-5\sqrt{2}\,t}$ $= -10te^{-5\sqrt{2}\,t}$ $\ddot{x} = -10e^{-5\sqrt{2}\,t} + 50\sqrt{2}te^{-5\sqrt{2}\,t}$ At Max $\dot{x}$ $\ddot{x} = 0 = -10e^{-5\sqrt{2}\,t} + 50\sqrt{2}te^{-5\sqrt{2}\,t}$ $5\sqrt{2}t = 1$ $t = \frac{1}{5\sqrt{2}} = \frac{\sqrt{2}}{10}$ $\dot{x} = -\sqrt{2}e^{-1} = -0.52026\ldots$ Max Speed = 0.5 m s^{-1}
	Obtains a correct equation to find the value of t at the maximum speed	1.1b	A1F	
	Obtains their correct t, any form	1.1b	A1F	
	Uses their value of t to obtain the velocity Must start from a damped harmonic model	3.4	M1	
	Obtains correct max speed, to at least 1sf or exact Condone missing units (–0.5 m s^{-1} = A0)	3.2a	A1	
	Total		**5**	

	Question total		**19**	

Q	Marking instructions	AO	Marks	Typical solution
12(a)	Obtains at least one correct non-zero argument/solution	1.1a	M1	$\cos 5\theta = 1$ $5\theta = 2n\pi$ $z = \cos 0 + \mathrm{i}\sin 0,$ $\cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5},$ $\cos\frac{4\pi}{5} + \mathrm{i}\sin\frac{4\pi}{5},$ $\cos\frac{6\pi}{5} + \mathrm{i}\sin\frac{6\pi}{5},$ $\cos\frac{8\pi}{5} + \mathrm{i}\sin\frac{8\pi}{5}$
	Obtains completely correct solutions must be $0 \le \theta < 2\pi$ (condone z=1)	1.1b	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
12(b)	States that all the points are the same distance from the origin	2.4	E1	Each of the solutions has modulus 1, and their arguments increase in steps of $2\pi/5$
	States that the angles between lines from the origin to adjacent points are all equal	2.1	E1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
12(c)	Expands $(\cos\theta + \mathrm{i}\sin\theta)^5$ Condone errors in or omissions of Imaginary part	1.1a	M1	$(\cos\theta + \mathrm{i}\sin\theta)^5 = 1$ Let $c = \cos\theta, s = \sin\theta$ $c^5 + 5c^4\mathrm{i}s - 10c^3s^2 - 10c^2\mathrm{i}s^3 + 5cs^4 + is^5 = 1$
	Obtains correct unsimplified Real part of expansion	1.1b	A1	Real parts: $c^5 - 10c^3s^2 + 5cs^4 = 1$ $c^5 - 10c^3(1-c^2) + 5c(1-c^2)^2 - 1 = 0$ $c^5 - 10c^3 + 10c^5 + 5c - 10c^3 + 5c^5 - 1 = 0$ $16c^5 - 20c^3 + 5c - 1 = 0$
	Equates real parts	3.1a	M1	as required
	Uses appropriate trig identity to express real part in terms of c	3.1a	M1	
	Completes a rigorous argument to obtain the required result	2.1	R1	
	Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
12(d)	Explains that z_3 is the complex conjugate of z_4 and that z_2 is the complex conjugate of z_5	2.4	E1	By symmetry $z_4^* = z_3$ so $\cos(\arg z_3) = \cos(\arg z_4) = a$ and $z_5^* = z_2$ so $\cos(\arg z_2) = \cos(\arg z_5) = b$
	Explains that the Real parts of the points on the diagram are the solutions of $16c^5 - 20c^3 + 5c - 1 = 0$	2.2a	M1	So $c = a$ and $c = b$ are both double roots of the equation $16c^5 - 20c^3 + 5c - 1 = 0$ and, by the factor theorem, $(c-a)(c-b)$ is a repeated quadratic factor of $16c^5 - 20c^3 + 5c - 1$
	Completes a rigorous argument to obtain the required result	2.1	R1	
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
12(e)	Deduces that h is a solution of the equation This may appear anywhere in the solution	2.2a	B1	h is a solution to the equation $$16c^5 - 20c^3 + 5c - 1 = 0$$ $$(c-1)(16c^4 + 16c^3 - 4c^2 - 4c + 1) = 0$$
	Factorises to obtain a linear factor and a quartic factor or better	3.1a	M1	Discard $c = 1$ $$(4c^2 + 2c - 1)^2 = 0$$ $$4c^2 + 2c - 1 = 0$$ $$c = \frac{-1 \pm \sqrt{5}}{4}$$
	Solves the quartic or quadratic equation correctly to get only two solutions	1.1b	A1	Select the solution with the greater absolute value; so
	Selects the correct solution	3.2a	E1	$$h = \left\lvert \frac{-1-\sqrt{5}}{4} \right\rvert = \frac{\sqrt{5}+1}{4}$$ as required
	Completes a rigorous argument to explain the required result	2.1	R1	
	Total		**5**	

	Question total		**17**	

	Paper total		**100**	